"EFFICIENCY" OF A PLATE FROM THE VIEWPOINT OF THE TRANSMISSIVE CAPACITIES OF THE MASS-TRANSFER STAGES

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UDC 66.012.7

The concept of the "efficiency" of a plate that is the ratio of the number of theoretical plates to the number of actual plates is analyzed. A dependence of the efficiency on the transmissive capacities of the mass-transfer stages is revealed. Expressions for analytical calculation of the plate efficiency in the case of linear approximation and different hypotheses on the structure of flows on a plate are obtained. It is shown that the efficiency can be larger than unity.

The traditional method of calculation of a plate mass exchanger, in particular, the determination of the number of actual plates, is based on the concept of plate "efficiency." Here the plate efficiency itself is determined experimentally or is calculated using empirical relations.

There are theoretical approaches to the determination of efficiency. As is shown by an analysis of the literature, in different references different concepts are meant by "plate efficiency." By definition, efficiency is the ratio of the number of theoretical plates to the number of actual plates [1, 2]. Nevertheless, other interpretations can often be met in the literature. For example, according to Murphy [2], efficiency is the ratio of the increments of component concentrations in a phase. And this efficiency, which is by no means equal to the ratio of the number of theoretical plates to the number of actual plates, is transferred to calculation of the number of plates without any reason at all. Naturally, this calculation cannot be called correct.

The aim of the present paper is to clarify the concept of efficiency and its use. We proceed from the definition of efficiency as the ratio of the number of theoretical plates to the number of actual plates. A theoretical plate is a part of the apparatus from which flows escape in equilibrium, which is possible only in the case where the mass-transfer surface in this part of the apparatus is infinite, i.e., on such a plate mass transfer occurs under the conditions of a "balance problem" [3, 4]. On an actual plate equilibrium of escaping flows cannot be achieved due precisely to the finite surface of mass transfer. Proceeding from these prerequisites, we consider a method of determination of the number of actual plates that allows for the actual surface of mass transfer on each plate.

Strictly speaking, use of the concept of efficiency is justified only when there is a more or less accurate description of the process. In this case efficiency takes into account small deviations of the real process from its theoretical description. We emphasize that here the values of efficiency differ slightly from unity. If the efficiencies of the process differ greatly from unity, then serious doubts are cast upon the suitability of the employed mathematical model for description of the process. In a number of mass transfer processes (e.g., absorption) the value of the efficiency does not exceed 5%. This suggests that in calculation of these processes in terms of the number of theoretical plates the main matter, namely, the mass-transfer surface, is disregarded, i.e., the model selected for description of the process (the model of theoretical plates) is indequate in this case.

Our model for theoretical determination of the number of actual plates is constructed on the transmissive capacities of the apparatus [3, 4]. All assumptions used as the basis for this paper correspond to assumptions (an equilibrium straight line, relative concentrations, constant fluxes of inerts, a constant coefficient of mass transfer) adopted in the development of the method of transmissive capacities. In accordance with this method, the plate

M. V. Lomonosov Moscow State Academy of Fine Chemical Technology. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 72, No. 2, pp. 323-328, March-April, 1999. Original article submitted October 14, 1997; revision submitted April 20, 1998.

1062-0125/99/7202-0303$22.00

column is considered as a number of apparatuses connected by a countercurrent. For this system we can write the dependence of the transmissive capacity of the column on the transmissive capacities of the stages of the mass-transfer process (supply, withdrawal, mass transfer through a phase interface) and the number of actual plates. Any scheme of motion of flows with this or that type of mixing can be realized on each plate. Altogether there can be 48 different schemes of motion of flows on a plate, called structural-configuration complexes [4]. For example, a straight flow, a crossflow, or other types of flow can be realized on the plate itself, and the phases are capable of moving in the mode of ideal displacement or ideal mixing or of having another flow structure.

We consider a mass-transfer process occurring in a counterflow plate apparatus with a number of plates n (or N_{act}) and mass-transfer surface on each plate f and on the whole apparatus $F = fn$. Counterflows of inerts D and W with an initial concentration of the transient component Y_{in} and X_{in}, respectively, are supplied to the apparatus. We assume for definiteness that the component passes from the Y phase to the X phase.

The kinetics of the mass-transfer process is determined by the coefficient of mass transfer, which is denoted by K_y in the "language" of the Y phase and by K_x in the "language" of the X phase, with $K_x/K_y \equiv R$, where R = const.

The total mass of the component passing from phase to phase in the whole apparatus is denoted by M. The concentrations of the transient component in the flows change from initial to final values: X_f and Y_f. The transferred mass is $M = D(Y_{in} - Y_f) = W(X_f - X_{in})$. Since the descriptions of the process in the languages of the phases X and Y are absolutely identical, in what follows we use the language of the X phase.

Using the apparatus of transmissive capacities for an analysis of this process, we write the transmissive capacities of the stages of supply, withdrawal, and mass transfer through the surface: DR, W, and K_xF, respectively. The transmissive capacity of the whole apparatus in the language of the X phase is M/Δ_x, where $\Delta_x = (Y_{in}/R - X_{in})$ is an initial difference of potentials.

It is not difficult to describe the mass transfer occurring on each plate by means of the apparatus of transmissive capacities. The transmissive capacities of the stages of supply and withdrawal will remain the same, since the plates in the column are connected in series. The transmissive capacities of the mass-transfer surface of each plate are K_xf. The transmissive capacity of an entire plate is (m/Δ_x), where m is the mass transferred on the plate and Δ_x is the difference of potentials at the plate inlet. Different masses m with their own difference of potentials at the inlet of each plate will be transferred on different plates, but the transmissive capacity of an entire plate (m/Δ_x) is the same for all plates of the apparatus. This is associated with the fact that the transmissive capacity of a plate depends only on the transmissive capacities of the stages and the structure of the flows on the plate, which are the same on all the plates [3, 4].

In the description of a plate it is often more convenient to use the ratios of the transmissive capacities rather than the transmissive capacities thermselves. We introduce the notation $a \equiv K_xf/(DR)$, $b \equiv K_xf/W$, $c \equiv DR/W$, $r \equiv (m/\Delta_x)/(K_xf)$. We emphasize that in the present paper the combinations a, b, and r refer to one plate.

The transmissive capacity of the column M/Δ_x is expressed by the formula [5]

$$M/\Delta_x = \frac{(1 - ra)^n - (1 - rb)^n}{nb\,(1 - ra)^n - na\,(1 - rb)^n} K_xF .$$

With account for the notation introduced above, this formula (in the "language" of the X phase) can be written in another form:

$$M/\Delta_x = \frac{(1 - (m/\Delta_x)/(DR))^n - (1 - (m/\Delta_x)/W)^n}{(1 - (m/\Delta_x)/(DR))^n/W - (1 - (m/\Delta_x)/W)^n/(DR)} .$$

After simple transformations we obtain a formula for the number of actual plates:

$$N_{act} \equiv n = \frac{\ln \dfrac{1 - (M/\Delta_x)/(DR)}{1 - (M/\Delta_x)/W}}{\ln \dfrac{1 - (m/\Delta_x)/(DR)}{1 - (m/\Delta_x)/W}}. \tag{1}$$

In order to determine the number of theoretical plates we need to find the limit of N_{act} for $K_x f \to \infty$. The value of N_{act} depends on $K_x f = K_x F/n$, since the transmissive capacity of a plate (m/Δ_x), involved in the denominator of (1), is, in turn, related to the transmissive capacity of the mass-transfer surface of this plate:

$$N_{theor} = \frac{\ln \dfrac{1 - (M/\Delta_x)/(DR)}{1 - (M/\Delta_x)/W}}{\lim\limits_{K_x f \to \infty} \ln \dfrac{1 - (M/\Delta_x)/(DR)}{1 - (M/\Delta_x)/W}}. \tag{2}$$

If necessary, we can write the formula for the plate efficiency in a general form:

$$\eta = \frac{N_{theor}}{N_{act}} = \frac{\ln \dfrac{1 - (m/\Delta_x)/(DR)}{1 - (m/\Delta_x)/W}}{\lim\limits_{K_x f \to \infty} \ln \dfrac{1 - (m/\Delta_x)/(DR)}{1 - (m/\Delta_x)/W}}. \tag{3}$$

As has been noted above, any of 48 structural-configuration complexes described by one of the eleven formulas of [4] can be realized on each plate.

If complete mixing occurs on a plate, then its transmissive capacity is written as [3, 4]

$$m/\Delta_x = \frac{1}{1 + b + a} K_x f = \frac{1}{1/W + 1/(DR) + 1/(K_x f)}.$$

For a theoretical plate with complete mixing of the flows the transmissive capacity has the form

$$\lim_{K_x f \to \infty} m/\Delta_x = \frac{1}{1/W + 1/(DR)} = \frac{DR}{c + 1}.$$

With allowance for the assumption of complete mixing of the phases on the plates the expression for N_{act} acquires the form

$$N_{act} = \frac{\ln \dfrac{1 - (M/\Delta_x)/(DR)}{1 - (M/\Delta_x)/W}}{\ln \left(\dfrac{W + K_x f}{DR + K_x f} \dfrac{DR}{W} \right)}. \tag{4}$$

In order to determine the number of theoretical plates, we need to obtain the limit of N_{act} for $K_x f \to \infty$:

$$N_{theor} = \frac{\ln \dfrac{1 - (M/\Delta_x)/(DR)}{1 - (M/\Delta_x)/W}}{\ln c}. \tag{5}$$

Then the expression for the plate efficiency is

$$\eta = \frac{N_{theor}}{N_{act}} = \frac{\ln \left(\dfrac{W + K_x f}{DR + K_x f} c \right)}{\ln c}. \tag{6}$$

The last formula can be used to determine $K_x f$ using experimental data in the case where the efficiency, W, and DR are known:

$$K_x f = W \frac{c^\eta - 1}{1 - c^{(\eta - 1)}} = DR \frac{c^\eta - 1}{c - c^\eta}.$$

If we assume that a straight flow with ideal displacement of the phases occurs on a plate, then the formula for determination of its transmissive capacity takes the form [4]

$$m/\Delta_x = \frac{1}{a + b}(1 - \exp(-(a + b)))\, K_x f$$

or

$$m/\Delta_x = \frac{1}{1/(DR) + 1/W}(1 - \exp(-(a + b))).$$

Then in the case $K_x f \to \infty$ the expression for the transmissive capacity of a theoretical plate is the same as for a theoretical plate with complete mixing of the phases:

$$\lim_{K_x f \to \infty} (m/\Delta_x) = \frac{1}{1/W + 1/(DR)}.$$

Having substituted the expressions for the transmissive capacity of a plate in Eqs. (1)-(3), we come to the corresponding formulas for N_{act}, N_{theor}, and η (in the case of a straight-flow plate). The number of actual plates is

$$N_{\text{act}} = \frac{\ln \dfrac{1 - (M/\Delta_x)/(DR)}{1 - (M/\Delta_x)/W}}{\ln \dfrac{DR + W \exp(-(a + b))}{W + DR \exp(-(a + b))}}. \qquad (7)$$

To determine the number of theoretical plates, we need to find the limit of N_{act} for $K_x f \to \infty$:

$$N_{\text{theor}} = \frac{\ln \dfrac{1 - (M/\Delta_x)/(DR)}{1 - (M/\Delta_x)/W}}{\ln c}. \qquad (8)$$

Hence we obtain an expression for the efficiency:

$$\eta = \frac{N_{\text{theor}}}{N_{\text{act}}} = \frac{\ln \dfrac{DR + W \exp(-(a + b))}{W + DR \exp(-(a + b))}}{\ln c}. \qquad (9)$$

The formulas for N_{theor} in the case of a straight-flow plate (ideal displacement) and a plate with complete mixing are, naturally, the same, because equilibrium of the flows leaving the plate corresponds to both of them. Correspondingly, the denominators of the formulas for the efficiency in these cases are the same. At the same time their numerators are different, they depend on the structure of the flows. Consequently, for different structures of the flows the effect of $K_x f$ on the efficiency is different. However, a calculation made with different sets of initial data showed only slight differences in the effect of $K_x f$ on the efficiency for different structures of the flows.

It should be noted that it is often more convenient to represent the numerator in formulas (1), (2), (4), (5), (7), (8) in a different form. Since $M = DR\,(Y_{\text{in}}/R - Y_{\text{f}}/R) = W(X_{\text{f}} - X_{\text{in}})$, after simple transformations the numerator can be represented as

$$\ln \frac{1-(M/\Delta_x)/(DR)}{1-(M/\Delta_x)/W} = \ln \frac{Y_f/R - X_{in}}{Y_{in}/R - X_f}.$$

A similar analysis can be made for cases where one of the flows is in the mode of ideal displacement and the other in the mode of complete mixing. There are two cases of this type, and the formulas for the transmissive capacity for them are [4]:

1) for the X phase (complete mixing) and the Y phase (ideal displacement)

$$m/\Delta_x = \frac{1-\exp(-a)}{a + b(1-\exp(-a))} K_x f;$$

2) for the X phase (ideal displacement) and the Y phase (complete mixing)

$$m/\Delta_x = \frac{1-\exp(-b)}{b + a(1-\exp(-b))} K_x f.$$

With these models results are obtained that are similar to the results for a straight-flow (ideal displacement) plate (7)-(9) and a plate with complete mixing (4)-(6). In the four models the denominators in the formulas for N_{theor} and η are equal to $\ln c$.

The picture becomes quite different if we assume that the plates in the column are countercurrent with ideal displacement of the flows. The transmissive capacity of a countercurrent plate is determined by the formula [4]

$$m/\Delta_x = \frac{\exp(-a)-\exp(-b)}{b\exp(-a) - a\exp(-b)} K_x f = \frac{\exp(-a)-\exp(-b)}{\exp(-a)/W - \exp(-b)/(DR)}.$$

For $K_x f \to \infty$ the transmissive capacity of the plate takes the form

$$\lim_{K_x f \to \infty} m/\Delta_x = \min(W, DR).$$

The formula for N_{act} with account for the expressions for the transmissive capacity of a countercurrent plate is written as follows:

$$N_{act} = \frac{\ln \frac{1-(M/\Delta_x)/(DR)}{1-(M/\Delta_x)/W}}{\ln \frac{\exp(-a)}{\exp(-b)}}. \tag{10}$$

To determine the number of theoretical plates N_{theor} and η we need to obtain the limit of N_{act} for $K_x f \to \infty$

$$N_{theor} = \frac{\ln \frac{1-(M/\Delta_x)/(DR)}{1-(M/\Delta_x)/W}}{\ln \frac{(DR - \min(DR, W))/(DR)}{(W - \min(DR, W))/W}}, \tag{11}$$

$$\eta = \frac{\ln \frac{\exp(-a)}{\exp(-b)}}{\ln \frac{(DR - \min(DR, W))/(DR)}{(W - \min(DR, W))/W}}. \tag{12}$$

In deriving formula (11) we used the same approach as earlier. It was assumed that a theoretical plate is a plate of the selected type for which $K_x f \to \infty$. Similarly, in the derivation of expression (12) for the efficiency, it was assumed that the latter is the ratio of the number of theoretical and actual plates *of one and the same type*, but for a theoretical plate $K_x f \to \infty$.

Having examined the values of N_{theor} and η, we can easily convince ourselves that these assumptions are inadmissible for the case of counterflow. These values are equal to zero because the denominator becomes either ln (0) or ln (∞). The point is that for infinite $K_x f$ a countercurrent plate cannot serve as a model of a theoretical plate. The flows leaving it are not equilibrium but lead the process away from equilibrium (i.e., the flows that leave will be on the side of the equilibrium line opposite the initial flows).

Thus, the plate efficiency should be considered as the ratio of the number of theoretical plates to the number of actual plates of the "selected" type. By a theoretical plate one should mean a plate of any type for which, with $K_x f$ tending to infinity, the transmissive capacity is

$$\lim_{K_x f \to \infty} m/\Delta_x = \frac{1}{1/W + 1/(DR)}.$$

This interpretation makes it possible to detemine N_{theor} and the efficiency for a countercurrent plate:

$$N_{theor} = \frac{\ln \dfrac{1 - (M/\Delta_x)/(DR)}{1 - (M/\Delta_x)/W}}{\ln c}, \quad \eta = \frac{\ln \dfrac{\exp(-a)}{\exp(-b)}}{\ln c}.$$

It should be noted that the efficiency for a countercurrent plate calculated in this way can take values larger than unity. We also note that an efficiency larger than one can be obtained for many cases of crossflow (formulas (3), (4), (6)-(9) in [4]).

We emphasize that the expressions obtained in the present paper are intended, foremost, for an analysis of the admissibility of using the concept of efficiency itself. In other words, we are dealing with the determination of the adequacy of using in calculations a model of a theoretical stage with subsequent passage to the real stage of mass transfer by means of the efficiency. And if the estimates show that the efficiency is unacceptably low, this should serve as grounds for:

1) rejection of the model of a theoretical stage and the efficiency as a basis for determination of the actual number of stages;

2) alteration of the technological situation (ranging from the design of the contact device to its operation) in order to increase the transmissive capacity of the surface stage of mass transfer, namely, its intensity and mass-transfer surface.

NOTATION

D, W, flows of inerts, kg/sec; F, f, mass-transfer surface of the whole apparatus and one plate, respectively, m^2; K, coefficient of mass transfer, kg/(m^2·sec); M, total mass of the component transferred in the apparatus, kg/sec; m, mass of the component transferred on one plate, kg/sec; N, n, number of plates; R, coefficient of distribution, kg/kg; X, Y, relative concentrations of the component in the phases, kg/kg; Δ, initial difference (of concentrations); η, "efficiency." Subscripts: act, actual; f, final; in, initial; theor, theoretical; x, y, the corresponding phase (flow).

REFERENCES

1. N. I. Gel'perin, Basic Processes and Apparatuses of Chemical Technology [in Russian], Moscow (1981).
2. A. G. Kasatkin, Basic Processes and Apparatuses of Chemical Technology [in Russian], Moscow (1973).
3. V. V. Zakharenko, T. I. Protskaya, V. V. Nosova, et al., Teor. Osnovy Khim. Tekhnol., 22, No. 3, 368 (1988).
4. V. V. Zakharenko and Yu. A. Malysheva, Teor. Osnovy Khim. Tekhnol., 26, No. 6, 812 (1992).
5. V. V. Zakharenko and A. L. Khandazhinskaya, "Transmissive capacity of a countercurrent circuit of appatuses," Deposited at VINITI, No. 1168-V96, 10.04.96.